Economy, probability and risk

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The science of probabilities has earned a special place because it tried through its concepts to build a bridge between theory and experimenting. As a formal notion which by definition does not lead to polemic, probability, nevertheless, meets a series of difficulties of interpretation whenever the probability must be applied to certain particular situations.

Usually, the economic literature brings into discussion two interpretations of the concept of probability: the objective interpretation often found under the name of frequency or statistical interpretation and the subjective or personal interpretation. Surprisingly, the third approach is excluded: the logical interpretation.

The purpose of the present paper is to study some aspects of the subjective and logical interpretation of the probability, as well as the implications in the economics.

Key words: objective, subjective and logical probability, uncertainty, risk

1. Introduction

As a personal opinion, we believe that we would be unfair if we would tackle this section by approaching those three meanings of the notion of probability, mentioned in the title, in an insensible manner. We could say that probability, as we understand it in every day language, is "the odds of a certain event to take place" or that it is, as the mathematicians define it, "a complete set function (numerable additive)". None of these statements would be inaccurate, but we would say too little about an outstanding concept which stands at the base of an inestimable theory.

The science of probabilities has earned a special place because it tried through its concepts to build a bridge between theory and experimenting. As a formal notion which by definition does not lead to polemic, probability, nevertheless, meets a series of difficulties of interpretation whenever the probability must be applied to certain particular situations. The French Maurice Fréchet believed that the reason of this conflict between the theory and the practice of probability consisted in the lack of a bridge between abstract and concrete and in the lack of a clear division between these two concepts. Even from the beginnings (1654) Pierre Fermat and Blaise Pascal, the founding fathers of the theory we are talking about, have built their arguments starting from perfect coins and dice, as if this thing would have been possible in reality. The famous Italian probability theoretician F. P. Cantelli shows that the development of the classical theory of probabilities has three stages[1]:

Stage I: The analysis of the empirical significations of the probability notions and equal-probable events notions, the empirical justification of the principles of the total and compound probabilities.

Stage II: The drawing up of the abstract theory (axiomatic) based on the principles of the total and compound probabilities. The theory that resulted from covering this stage was independent from the "physical" notion of probability.

Stage III: Confirming the arrangement between the formal principles and the experimental actions. Only in this stage one may use the probabilities theoretically obtained for the purpose of anticipating the relative frequencies of the regarded events.

Even from reading the first stage we have reasons to intuit that the nature of an event or of an empirical situation may cover the concept of probability in a specific aura. Moreover, the academician Octav Onicescu[2] states explicitly this possibility:

"The probability (…) for the types of concrete events of the experiences has a significance which varies with the experience. As the substance of the elements which makes the object of mechanics varies from element to element and we do not keep in mind for the mechanic model only the mass value, which is a number resulted from a measurement, so it is the probability which leaves for the probabilistic model only the numerical value, holding for another research which overtakes the one given by the probabilistic model, the problem of its own specific significance within the considered experience."

Therefore, we expect the possibility of the existence of some interpretations to have a common ground in the formal, axiomatic theory. In order to avoid any misunderstandings, we must specify that the problems do not appear within the model itself which has an incontestable, abstract, mathematical nature, but they appear only within

expressing the interpretations and the way these are put in straight relation with the formal model.

Usually, the economic literature brings into discussion two interpretations of the concept of probability: the objective interpretation often found under the name of frequency or statistical interpretation and the subjective or personal interpretation. Surprisingly, the third approach is excluded: the logical interpretation. We say surprisingly because its existence is owed to Keynes (1921) whose contributions to the theory of probability are not known by too many economists. Getting here, some readers may become malicious insinuating that it took the intervention of an economist in order to bring a "logical" approach to the notion of probability. As soon as we begin to describe these three interpretations the things will clear out, but not before we say that "interpretation" does not mean redefining the concept, that "logical" means something in the spirit of mathematics' logic and that the objective and subjective approaches have nothing "illogical".

2. Probability between objective, logical and subjective

The objective, frequency or statistical interpretation to which we usually refer regarding the objective probability and which gives us in the practical applications the psychical balance provided by a scientifically support of the information we operate with, is based on a law of random. This states that the frequencies in the appearance of a random event, in a multitude of recurrences (under identical conditions) of the experience are grouped around a central value. Thus the objective probability becomes the limit's value (in a well specified way) of the relative frequencies the event appears. In order to give some examples we shall refer to the data from Table 1 where the following are registered: how many times a coin has been flipped, how many times the tails appears and the relative frequency calculated as a proportion between the number of appearances and the number of their recurrences.

The number of flipping,	The number of "tails" appearances,	Relative frequency, m/n
4040	2048	0,5080
12000	6019	0,5016
24000	1212	0,5005

Table 1. :*The relative frequency obtained by researchers Buffon and Pearson, to the appearance of the "tails" side on the repeated flipping of a coin*

From Table 1 one may notice that according as the number of repeating the experiment grows, the relative frequency of the appearance of the "tails" side groups around the 0.5 value, which we all admit to be the probability (the objective one) that when flipping a coin to obtain any of those two sides. The objective nature we all got used to allot to this chance is also due to the fact that the 0.5 value is independent from the personal opinions of the observer of the experience.

The logical interpretation, as we mentioned before, was first expressed by J. M. Keynes in 1921. Although it was expressed abstractly, it has been from the very beginning extremely promising and it had as a natural result the emergence of the probabilistic logic. In Picture 1 we depicted for a better understanding of this interpretation, the following elements:

1. An initial statement, E, which in practice may play the role of the empirical data of which we own in a certain context.

2. The set of statements, $E_1,E_2,...,E_n$ afferent to various hypothesis which may be advanced based on the statement or on the initial data, E.

3. The set of real numbers, $p_1,p_2,...,p_n$ attributed uniquely to each correspondence $E \to E_i,\ i = \overline{1,n}$

$$\langle StatementE \rangle : \begin{cases} \xrightarrow{p_1} Hypothesis\ E_1 \\ \xrightarrow{p_2} HypothesisE_2 \\ ..... \\ \xrightarrow{p_n} Hypothesis\ E_n \end{cases}$$

Figure 1. *Depiction of a logical interpretation of the probability*

In this interpretation, the real numbers $p_1,p_2,...,p_n$ represent degrees of **confirmation** of the hypothesis $E_1,E_2,...,E_n$ based on the initial empirical data. There is only one step left from this approach to the construction of what we call nowadays the distribution of probability of a random variable.

The subjective interpretation of the probability becomes easier to explain once we have explained the logical interpretation. In a depiction as that on Picture 1., the values $p_1,p_2,...,p_n$ do not represent a confirmation degree this time, but a trust degree so, therefore they are not uniquely determined anymore. The subjective interpretation does not regard the probability as a logical relation between the initial data and the formulated hypothesis anymore, but as a quasi-logical relation, the difference being that which can be perceived

between the elements used in the description: confirmation versus trust. The probability values are still included between 0 and 1, but they may vary proportional to mentality, personality, the information of the person involved in observing the relation between the existent data and the formulated hypothesis based on these values.

For the purposes of the present paper, we believe that it is very important the thorough study of the subjective interpretation of the probability. In many papers which deal with the risk matter, the authors state the fact that the lack of objective probabilities and their replacement with subjective probabilities makes it impossible to rationally choose between alternatives[3]. This is the reason why the subjective probability is destined only to an attempt of decreasing the uncertainty and in no way does it seem to be involved in the matter of assessing the risk. This statement becomes, for many economists, an unwritten law which gives the subjective probability the palliative position when fighting against risk, without granting too many privileges with scientifically values. It is true that things have a certain doses of correctness, but the approach we find in economical literature regarding the subjective probability is definitely unfair and the lack of information in the field makes a lot of people exclude an important weapon from the arsenal destined to confronting the risk. We must be aware of the fact that in practice we rarely have in handy enough data in order to be able to build an objective probability based on the relative frequencies method. Therefore, the nature of most probability values which we calculate is a "relatively objective" one, to be more precise a "sort of subjective" one, because we fill in those inexistent information, in a personal manner, with personal beliefs.

Following the same reasoning, would mean that no other estimation of a probability should be labeled as being indubitable, as well as the results of the actions and decisions taken based on the estimated values should not be convenient, except for the time when Providence interferes. We can realize that in reality things are not like this, because many forecasts are sufficiently precise, because many evaluations are in accordance to reality and because powerful institutions which are grounded on the calculation of the probabilities are operating and developing unhampered. Is there something in the "subjective" problems which slips the majority? Certainly yes, although it shouldn't be: the things have been made clear since 1935, so there has been plenty of time to clear out the problems. It may be possible that many times the tradition is more powerful than the information and this would somehow explain why too many economists do not grant the deserved credit to the subjective probability.

As soon as we can accept the fact that the subjective interpretation of the probability is a logical theory, our

attitude may change. The logic must consist in the fact that there are only some combinations of trust degrees which may be accepted, to be more precise, if one person has a certain trust degree p in a statement then this person in a complementary statement should have the trust degree 1-p. Picture 2 displays this requisite which must be carried out from the very beginning, in order to be able to say that the subjective interpretation is a logical theory.

$$\langle Statement\ E \rangle : \begin{cases} \xrightarrow{p} Hypothesis\ I \\ \xrightarrow{1-p} Hypothesis\ \ non\ I \end{cases}$$

Figure : *The condition of coherence for the subjective interpretation of the probability*

In other words, P(I)+P(nonI)=1, relation which actually expresses one of the fundamental rules of the probability theory. This characteristic is named "the condition of coherence" and may be easily generalized, meaning that starting from the initial data E, an individual grants the trust degrees $p_1,p_2,...,p_n$ to those n hypothesis $E_1,E_2,...,E_n$ which he sees as being possible, then $p_1+p_2+...+p_n=1$.

At first sight, the condition of coherence may seem purely theoretical and having no relation to the risk matter. Immediately two questions arise:

1. Why an individual must be "coherent" when granting trust degrees?

2. What is the connection between the coherence relation and the risk?

First of all, we shall answer the first question and we shall say that the individual has no obligation. Even though the common sense tells us that when an event has 50% chances of producing, it must also have 50% chances of not showing up, no one can stop someone from granting, for instance, the trust degrees 50% and respectively 75% for those two complementary possibilities. Everyone is free to believe what he wishes, but it is interesting to observe what may result from supporting this kind of ideas.

Let's say we confront an event about which a person strongly claims that it has the following probability values

P (event's occurrence) = 0.5

P (event's non-occurrence) = 0.75

For our own sake, we suggest the following options to this individual: to bet 50 € on the occurrence of such event and only 25 € on the second version, which seems a little more certain for the individual. Here is the result of accepting the bets:

• **The version "event's occurrence"**: we lose 25 € put on the non-occurrence of the event, we receive back the 50 € from the first bet and we win 50 € from the individual, because the chances were 1:1.

• **The version "event's non-occurrence"**: we lose 50 € put on the possibility of the event's occurrence, we recover the 25 € invested and we receive 3x25 = 75 € from the individual, grounded on the fact that the winning chances in this last version (75%) were of 3:1.

In both situations we invested 75 € and from both situations we have the certainty of obtaining 100 €. Therefore, the lack of coherence of the individual grants us the possibility of obtaining a profit without a risk and it grants him the possibility of certain loss. Thus, more than a theoretical condition needed in order to ground the mathematical results the coherence becomes a behavior recommendation. Once we accept this, we can go on in supporting the logic of subjective interpretation.

Although you decided to agree with the representation in figure 2, you will certainly disagree with the message in figure 3.

$$\text{Flipping a coin} \Rightarrow \begin{cases} \xrightarrow{p=0,2} \textit{Tails} \\ \xrightarrow{1-p=0,8} \textit{Heads} \end{cases}$$

Figure 3: *Degrees of trust in the outcome of flipping a coin*

One's revolt when seeing such supposition might seem grounded, because even a child knows that on each of the two rows the (objective) value 0.5 should appear. Well, that person does not know or suspects a fake coin, and that is why he/she has chosen these surprising ranks of confidence. No problem up to here, but they could appear as soon as our character would start acting and making important decisions grounded on his/her beliefs. And because people have a special tendency towards this sort of initiative, the persecution occurred of subjectively interpreting the probability. Actually, if our suspicious individual would be open to carefully notice enough repetition in the coin flipping, in time he/she would reach the conclusion that he/she "thought" wrong. With each repetition of the experience, the observer gathers information which corrects the initial beliefs, so that his/her opinions gradually become normal. Actually, the

individual would apply in this way the procedure of classic statistical inference of Bayesian style, ascertaining that starting from an initial repartition of probabilities, such are modified consequent to and grounded on the experimental data obtained. This is where the most important concept of subjective interpretation intervenes, which was called by De Finetti equivalence or symmetry and which made possible the connection between subjective interpretation and Bayesian inference. The conclusion of this concept's existence is stated as it follows[4]: for equivalent events, irrespective of the initial combination of trust degrees, the outcomes of applying the statistical inference are the same: the same decision to be made, the same hypothesis to be accepted, the same (or almost the same) value of an undertaken parameter. In order for our explanations to be complete, and in order for the reader to be persuaded of the possibility of improving the trust degrees until the objective probability is obtained, we need to underline that the information were mathematically proven in 1931 by Bruno de Finetti, who showed that after a sufficiently large number of observations, an individual, regardless his initial trust degree, should allocate to the events taken into account trust degree close to the relative frequencies.

The conclusion which takes shape is that it does not matter too much what we initially think, because our attention, our ability of analyzing and synthesizing information we obtain throughout the process are enough in order to achieve objective probability values. Of course, for an economist such insurance does not represent an instantaneous settlement of all the conflicts between objective and subjective and such view is a grounded one. Obtaining information, analyzing and processing it are time consuming activities, and such time is something that the enterpriser, in his fight against risk usually does not have. This is the reason for which the next chapters shall provide methods and techniques for identifying and assessing the effects of those factors affecting the company.

Some readers might say that we insisted very much on the interpretation of probabilities. For that it is to be blamed our conviction that a science, such as the one regarding the risk cannot be grounded without a thorough understanding of the fundamental concepts. We could have focused on a formal presentation, based on the axioms in the probability theory, but such approach would not have served our purpose of knowing the level of the random extent in view of its various interpretations. Moreover, the mathematical side of the probabilities is familiar to and understood by the specialists in the risk area, but a very important aspect is being ignored in the specialized literature: the correct distinction between the subjective probability and the objective one.

3. Uncertainty and risk: conceptual incorporation and delimitation

This is the moment when we focus on the actual implications generated in the environment by the already accepted existence of probability, the moment when we make the transition towards the economical approach, towards the subject of the present paper.

When we say "economy", we refer to a very complex collection in which activities are being carried out, aimed to meet our needs. Whether we refer to producing and trading goods or services, or to providing information, obtaining, distributing or re-distributing incomes, saving tools or means of individual or group insurances, "economy" is the core of our very social life. Each individual can claim that he is unique and that his needs have a particular feature. But physicists teach us that in nature there are no isolate systems, and that every action we take is closely connected with numerous other initiatives and has multiple effects. Therefore, even if sometimes we need to refer to a micro, macro or global economy, we must not omit that inside and between such levels there are numerous connections. One's daily experience proves that economy and random are indissolubly connected notions, and it would be useless to try to give detailed reasons for such assertion, because the obviousness is overwhelming.

Also, there is no news that every voluntary human action is grounded on a prior decision. The decision is made depending on the way in which the individual has perceived the phenomena on which he is to decide, and perception is directly dependant on the specific features of the one making the decision. We rarely know from the beginning whether the alternative we choose is the best one, but we try to make decisions grounded on our knowledge and hoping that our choices are correct. In its turn, each economical activity implies successive choices affecting both the decision maker and many other persons; that is why the formal theories aiming this direction become more and more scientifically grounded. Under such circumstances, there is no surprise that approaching the random has new implications, and that the close analysis of this element of our current life should become an ever present component in the life of each economical sector.

Problems always occur on the line between "must" and "do", generated by a multitude of opinions on "how to do". The polemics arising from such can be almost hilarious, sometimes having the effect of denying "must". There is no doubt that in order to manage the random it must be previously measured, let aside the fact that before measuring it its right of existence must be admitted. And we are not referring here to the existence of random itself, of which we just said that we had no doubts, but to the existence of random in the life of the economical entity.

Although it might seem surprising these days, the economical phenomena

have not been always approached in view of the unexpected. Before the occurrence of the probabilistic economy (1942) due to the Norwegian professor T. Haavelmo, the economical theory implied a deterministic path for the economical phenomena and behaviors, and the scientists of those times thought there was complete similarity between the forces inside the mechanic physical systems. Due to the fact that they were holding on the term "rational", the idea was unacceptable that an economical entity could for a second undertake an action having a random result. Probably such belief is not essentially wrong, if we view it from the stand point of the cause-effect axiom. Blaise Pascal believed, as we do, that all causes influencing an event precisely determine together the development of that event, because the same causes have always the same effects[5]. With such belief, it seems odd that this scientist was the father of the probability theory and that he made the following assertion: "he who admits the cause-effect axiom must also accept the axiom stating that random events have a probability which is independent from us, and thus objective, because this is nothing but a more precise stating of the same fundamental principle". We dare to say that if these two propositions were not perceived as contradictory, the random would have been included in the economical theory much sooner. The uncertainty and risk could have been delimited simpler and the probabilistic economy would have had its influences much earlier among the theoreticians... Instead, the development of the two concepts was shaken[6] and the polemics on their definition still continues nowadays.

Let us discuss briefly on the assertions of Pascal, for the purpose of extracting useful information on the description of uncertainty and risk. There is no doubt that the principle of cause-effect depicts a deterministic connection between the cause or a complex of causes and the development of the event up to the obtaining of the outcome. Figure 4 describes this principle in graphical form.

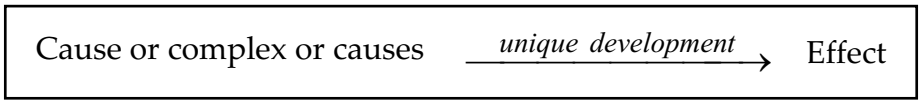

Figure 4: *Cause-effect axiom, in graphical form*

By an apparent contrast, the second proposition, which is actually identical to this one, renders a non-determinist vision, stating that random events exist actually – although based on the cause-effect principle they would not exist – and that all these are described by objective probabilities, independent

from the observer. The first ones who will feel damaged by the idea of independence will be the adepts of quantum mechanic, who will assert that by simply observing the system, the observer does not change its status.

Despite some so-called obviousness, Pascal supported an un-doubtful truth. Let us present a rephrasing of the cause-effect principle, stating that all the conditions which can influence a certain event have a unique effect over the development of that phenomenon. With a little modesty and a lot of realism, we must accept that we cannot know all the conditions, nor can we be aware of them – nature is not random, but if we perceive only part of all the conditions, and this is usually the case, then from our stand point the development of the phenomenon does not have a determined path, but more possibilities are certain to appear and for each of them a certain probability. The random event reserves its right of taking place or not, because the incomplete knowledge over the beliefs makes the result to not be unique, determinate; determined is the chance of occurring closely regarding that part of the conditions we manage do perceive. Regardless whether we succeed in determining the probability values or not, they exist and have nothing to do with our disputes regarding the uncertainty and risk. In measuring these values, it is not important if we do not know or if we cannot know the numbers which ought to be attributed, as less important as the attitude of an individual who asks himself whether he does not how or he cannot be a swimming world champion. The distinction between risk and non-risk is the distinction between the following propositions:

A we have the probability values
non A: we do not have the probability values

To not have may mean not to know of not being able to know, but the difference between the two possibilities is already the problem of the individual. Nature, in its wonderful arrogance, is not interested at all of such distinction.

The need for order makes us try to put a label on the situations in which we are. Figure 2 uses a tree diagram in order to delimit the three major concepts, namely: non-determination, uncertainty and risk, which we are to note herein by N, U and R. Grounded on such representation, we can describe non-determination as being the most severe form of random, in which the results of an experience are not known and therefore the matter of assessing their probabilities is senseless. The uncertainty is a version somehow more comfortable, in which at least we know what to expect, even if we cannot know the probabilities of outcome occurrence and/or their magnitude. At last, risk seems to be by far the most convenient situation from this entire picture, because an experience is governed by risk if its results are known, so are the probabilities and the magnitude of such.

$$\textit{Random experience}\begin{cases}\rightarrow \textit{Known results}\begin{cases}\rightarrow \textit{Unknown results} \rightarrow N\\ \rightarrow \textit{Unknown probabilities} \rightarrow U\\ \textit{Known probabilities} \rightarrow \begin{cases}\textit{Unknown magnitudes} \rightarrow U\\ \textit{Known magnitudes} \rightarrow R\end{cases}\end{cases}\end{cases}$$

Figure 5: *Description of non-determination, uncertainty and risk by means of the tree diagram*

The delimitation we made in figure 5 is not obviously our creation, but it is the result of a cumulus of information we have obtained by studying the opinions of various authors. It is true that sometimes the specialists may have contradictory visions: what represents a risk for some is uncertainty for others and vice-versa; the non-determination is many times left out of the discussion; however, the majority opinion is the one expressed in the graphic form above. We find the need to say that the terms of risk, uncertainty and non-determination by themselves have no value. If instead of them we used the notations A, B, C, then we should take into account the hypothesis implied by such labels and not the semantic interpretations of the three letters. We also find it relevant to make the clarifications that we have not expressed at any time our intent of defining these three concepts, but only of characterizing them. The attempts of defining such make object to complex polemics, of more than 100 years old and it was inevitable to question which the reason was. Strictly speaking, the definition must be a correct, complete answer, of comprehensive nature, in which any omission or inclusion changes the defined object.

4. Conclusions

The risk, the uncertainty and the non-determination have not been characterized by any sort of statement. There is a vast literature on this area, but rarely two authors reach the same conclusion on a so-called definition of terms. Therefore, we can say that these three concepts have been described so far in numerous ways, that numerous features of them have been emphasized, which characterized them depending on the conjuncture, but unanimous accepted definition have not been enunciated, as it is for example the definition of demand for a product. All this disturbing agitation tempted us to believe that risk, uncertainty and non-determination should be primary notions similar to the mathematical terms of set, thus being unable to define them, but only to describe them by means of the manifestation forms and their effects. Consequently, our refined readers shall forgive us if this paper does not reopen the polemic of definitions, and we dare to believe that similar to the way in which the set theory was set up on an un-defined concept, our paper can comprise coherent assertion without undertaking a new attempt of defining the risk. We hereby end this paper

saying that we use to refer to risk as that random phenomenon potentially generating damages, which is characterized by the possibility of establishing the results, the objective probabilities and the magnitude of the impact over the affected object. We also need to say that the momentarily inability of knowing objective probability values must not demobilize an individual in applying the methods and techniques which are particular to risk management. One only needs that taking into account the results of the Bayesian inference, to gather information which would allow him to adjust the probabilities up to obtaining results close enough to the objective values.

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