# Hertzian Dynamic Models In Ludwig Wittgenstein's Theory Of Logic[1]

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***Abstract:*** *During the last century the social sciences grew from the stage of speculative system building to a more mature development in which empirical data are sought for the significance they can have for systematic theories. A lot of work in this field concerns itself with determining the methodological and conceptual prerequisites for a mature science of human reasoning and behavior. Modeling human reasoning and human behavior, although currently focused on social and economic phenomena like organizations, organizational knowledge, leadership, cooperation etc., are historically dependent upon modeling natural phenomena in physical science, precisely because physical science tackles successfully the issue of building upon empirical data. This paper pursues an apparently small, but nonetheless significant, historical claim concerning the "relative position of human reasoning and mechanics" a claim made possible by the development of late 19th century's epistemology of science (mainly Heinrich Hertz's) and theoretical philosophy (Ludwig Wittgenstein). The main idea of the paper is that Ludwig Wittgenstein's concept of "logical representation", seen as a landmark for what human reasoning is about, is an intricate analogue to the Hertzian "dynamic models" from the Principles of Mechanics. This analogy is analyzed and explained with regard to the problem of the "logic of color".*

***Key words:*** dynamic models; human reasoning; Hertzian mechanics; Wittgenstein' logic of color, representation.

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**1.Introduction**

Almost every specialist in social sciences (and especially in economics) is familiar with game-theory, a very strong logico-mathematical tool used to model human behavior: e.g. cooperative behavior, organizations' dynamics, the emergence of social groups and group norms, the emergence and features of leadership based on cooperative/ defective behavior etc. This tool is also effective in studying human reasoning and cognitive biases from an evolutionary perspective and that is why it fits well in the analysis of group phenomena especially when the dynamics of such groups becomes significant for economics or for sociology. But the tradition of using logical or mathematical modeling, in order to capture human reasoning and human action in various practical contexts that undergo aggregation and change, spreads from a much older root. In fact, logico-mathematical modeling is historically and conceptually connected with the activity of modeling natural phenomena in physical science. This practice is made explicit, for instance, in the development of the late 19th century's epistemology of science (H. Hertz, L. Boltzmann), and it relies on the concept of "dynamic models" from classical mechanics and on several philosophical developments regarding the concept of human reasoning as an activity of manipulating and integrating "representations" (L. Wittgenstein).

The conceptual interplay between "representations", "models" and "dynamics" appears in the writings of several important nineteenth century scientists and epistemologists of science such as Heinrich Hertz, Hermann von Helmholtz and Ludwig Boltzmann. Hermann von Helmholtz, for instance, is the author of a theory of perceptions as projections on abstract n-dimensional manifolds that allow for perceptual reasoning as dynamic transformations on these manifolds, while Ludwig Boltzmann is credited with the idea that the language of science is an integrating image (Bild) of physical reality. Starting from here a large number of commentators and interpreters (P. Baker, P.M.S. Hacker, N.Griffin, E. Stenius, E. Anscombe etc.) of Ludwig Wittgenstein's early philosophical writings have suggested different hypotheses regarding the signification of "images" (Bilder) in his philosophical work Tractatus logico-philosophicus. One interpretation (Visser, 1999) associates early Wittgenstein's account of propositions as images with von Helmholtz's phenomenological analysis of perception. Thus, the proposition as an "image" (Bild) of reality is the expression of a Vorstellung (or inner representation) and it is composed of elementary sense-data. By contrast, other interpretations (Hyder, 2002; Hamilton, 2002), relate Wittgenstein's thesis that propositions are images with a physicalist perspective on representation. According to such points of view, propositional images are expressions of material representations of reality (Darstellungen) that form independently from the mind and are composed of real objects (like physical atoms). It is significant to note that the distinction between these two kinds of representations – inner (Vorstellungen) and material representations (Darstellungen) – was used by Ludwig Boltzmann to distinguish science from phenomenological epistemology. In his view, material representations of reality were generated by the laws of physics and they satisfied certain conditions of validity that inner representations could not satisfy.

If we take a close look at Tractatus logico-philosophicus, it is quite obvious that Ludwig Wittgenstein is not referring to physical atoms, nor to elementary sense-data (meant to build up a subjective experience of reality) when speaking about Bilder, but to a generic notion of atom involved in human reasoning. My thesis is that it might be insightful to pay attention to Heinrich Hertz's theory of dynamic models if we want to fully understand Wittgenstein's idea that the linguistic entities (i.e. propositions) involved in human reasoning are "images" (Bilder) made of logical atoms.

In his Mechanics, the physicist Heinrich Hertz talks about a geometrical representation of nature. His point of view is, in some respects, perplexing. Hertz refers to his models both in terms of "mental images" (Scheinbilder) and of "physical representations" (Darstellungen) of nature by means of physical laws. However, his choices of words and concepts may lead the researcher astray. It is a fact that in the late nineteenth century physics and epistemology of science there was no unanimous consensus regarding to what a model of nature really is. This may explain some of Hertz's ambiguities, but also blur some of his theoretical intentions. That is why I shall bring into focus a more specific and technical distinction that could give a clearer meaning to Hertz's conception of models. What I have in mind is the distinction between the "scenario" of a dynamic model (a concept that would explain Hertz's notion of Scheinbild) and the "parameterized representation" of a physical phenomenon within a dynamic model that would correspond to Hertz's notion of Darstellung.

The most important feature of a dynamic model is that it follows the temporal evolution of a phenomenon (like motion, for instance) by extracting physical consequences from an initial state – described, in principle, by a bunch of partial information. A dynamic model allows us to derive correctly future states of a physical system even though we do not have a complete representation of the initial state of that system. A good example would be the differential representation of motion – where masses and forces that act upon material points are neglected. However, the partial information needed has to be organized in a relevant manner in order to allow for the extraction of desired consequences (for instance, time and position must be considered independently). This incomplete, but organized, information is what it is called the "scenario" of a model. It is interesting to see that in his introduction to the Principles of Mechanics, Heinrich Hertz is using a very similar concept, Scheinbilder:

"We form for ourselves images [innere Scheinbilder] or symbols of external objects; and the form which we give them is such that necessary consequents of the images in thought are always images of the necessary consequents in nature of the things pictured." (Hertz, 2001, 1).

A "scenario" is also abstract – it structures information into relevant entities, properties and relations. For instance, in the differential representation of motion we have abstract material points characterized by independent abstract properties like spatial and temporal position (in a motion space). So Heinrich Hertz seems to refer to some abstract information about a state of a system, relevant to the extraction of necessary physical consequences (or reliable predictions). But in order to make reliable predictions, a model needs, along with an adequate organization of initial information, some equations of condition that allow for a perspicuous calculation of observable quantities such as displacement. It is worthwhile to stress that in Hertz's dynamic models mass is introduced in the motion space not as an independent variable, but as a parameter built in the equations of condition. This suggests that Heinrich Hertz had in mind a peculiar representation of motion and a different organization of motion "scenarios" from the standard cinematic representation. Indeed, he seems to propose a "parameterized representation" of displacement or a Darstellung, through equations of condition with build-in parameters. This peculiar approach to models (Modelle) gives a significant and technical load to the notion of Darstellung that is not present, for instance, in Boltzmann's account, and yet, as we are about to see in the following sections, of great relevance to Ludwig Wittgenstein's atomism from Tractatus logico-philosophicus.

## 2. Dynamic models

Wittgenstein's references to Hertz's dynamic models in the Tractatus are rather scarce. In fact, the only explicit reference is to be found at 4.04:

"In a proposition there must be exactly as many distinguishable parts as in the situation that it represents. The two must possess the same logical (mathematical) multiplicity. (Compare Hertz's Mechanics on dynamical models.)"

This passage brings into focus two related issues: first, the connection between propositional "parts" or elements and logical multiplicity (Mannifältihkeit), and second, the correlation between logical multiplicity and models. It is relevant to stress that in the secondary literature there are loads of studies that concentrate on the first issue, while to the second the references are rather few (Barker, 1980; Grasshoff, 1998; Tougas, 199;, Lammpert 2000). Usually, the concept of logical multiplicity (Mannifältihkeit) is considered a terminological influence from Hertz, with no substantial connection to the theory of dynamic models presented in his Principles of Mechanics. In the following, I shall argue that Wittgenstein's concept of Mannifältihkeit bears indeed a substantial connection to the theory of dynamic models from Hertzian mechanics, and that this connection is important in order to give a proper account of early Wittgenstein's atomism.

But first let us see what logical multiplicity (Mannifältihkeit) means. I shall keep certain remarks quite brief because it is impossible to give an extensive account in such a short paper. As some interpreters have already pointed out, Hertz's concept of Mannifältihkeit is an important influence from Riemannian geometry, absorbed through von Helmholtz's theory of perception (Hyder, 2002):

„Riemann calls a system of differences in which the individual element can be determined by n measurements, a n-fold manifold, or a manifold of n dimensions. Thus the space that we know and in which we live is a three-fold extended manifold, a plane a two-fold, and a line a one-fold manifold, as is indeed time. The system of colors also constitutes a three-fold manifold, in that each color can be represented… as a mixture of three elementary colors, of each of which a definite quantum is to be chosen… we could just as well describe the domain of simple tones as a manifold of two dimensions, if we are to take them to be differentiated only by pitch and volume." [1]

According to von Helmholtz all our empirical knowledge is organized in complexes of elementary data, called manifolds. Colors, sounds, time etc. are such manifolds. The basic idea borrowed from Riemann's geometry is that any quantity can be defined as a point in a space of n-dimensional measurements. Of course, in order to get an adequate measurement of the desired quantities, it is necessary to determine the right multiplicity of the magnitude space, i.e. the correct number of dimensions. In this respect, von Helmholtz uses a phenomenological device – i.e. how colors, sounds, time etc. form in human perception. Thus, he observes that colors can be analyzed 3-dimensionally as mixtures of three elementary colors; sounds can by analyzed 2-dimensionally by measuring pitch and intensity; time is a 1-dimensional object of our inner perception etc. So multiplicity is determined phenomenologically and this seems to be one of the key aspects of Herman von Helmholtz's epistemology. His manifolds are perceptual manifolds. Interestingly, they seem to offer a good account of Ludwig Boltzmann's concept of Vorstellung. As we have seen in the first part, a Vorstellung is a sort of internal model of reality. Perceptual manifolds are in von Helmholtz's epistemology internal models of reality – with adequate multiplicity.

Keeping these observations in mind, we could see by analogy what the author of the Tractatus meant by his concept of logical multiplicity. Let us start with a simple example, a sentence like: "This stick is 1.5 meters long". We may ask ourselves now what does it mean for a stick to be 1.5 meters long? It seems that in order to be able to talk about length we need a system of measurements for length, such as a yardstick, i.e., a 1-dimensional manifold. Without the one-fold of length, the sentence "This stick is 1.5 meters long" would not have any meaning. Likewise, any sentence bears with it a system of logical "measurements": we know, for instance, when a sentence refers to an object, to a property, a relation etc. We may spot easily such differences as between "John is in the yard" and "Yellow is brighter than gray", although the mechanism of such differences is by far more intricate than in the case of length. To such differences was intended to answer, for instance, Betrand Russell's theory of types. However, never convinced by Russell's theory, Ludwig Wittgenstein chose in the Tractatus a different solution, i.e. to deal away with predicative and relational concepts and present the system of logical differences in a quite original manner: the projection (Abbildung) of sentences like "John is in the yard" and "Yellow is brighter than gray" on an aggregate of logical manifolds (called logical space).

The logical measurements (that give the logical multiplicity of a situation like John's presence in the yard) are introduced in language along with each sentence like lengths are introduced along with each quantity-expression: "1 meter", "2 meters" etc., and therefore each sentence is associated with a manifold model, called by the author of the Tractatus, Darstellung. Later in Philosophical Remarks, Ludwig Wittgenstein recalls this approach of associating sentences and situations with the idea of logical manifolds:

"When I built language up by using a coordinate system for representing a state of affairs in space, I introduced into language an

[1] (Hyder, 2002, 26).

element which it doesn't normally use. This device is surely permissible. And it shows the connection between language and reality. The written sign without the coordinate system is senseless." [2]

But, in order for this approach to work, it is necessary to express correctly the multiplicity of the situation (i.e. its correct number of dimensions). As mentioned before, Hermann von Helmholtz used a phenomenological device: how qualities decompose in perception. Although Wittgenstein often refers (especially in "Some Remarks on Logical Form" and in Philosophical Remarks) to a color and a sound space and even to the visual field as a substitution for physical space when presenting his examples, he does not seem to have in mind a Helmholtzian phenomenological reduction, but only some pertinent analogies. His statement concerning multiplicity (Mannifältihkeit) is referring to propositional and factual "parts" that can be depicted via a projection on coordinate systems corresponding to logical properties. A clearer image of this perspective can be found in Betrand Russell's lecture on logical atomism from 1924 (three years after the publication of Tractatus logico-philosophicus):

"When some set of supposed entities has neat logical properties, it turns out, in a great many instances, that the supposed entities can be replaced by purely logical structures without altering in any detail any of the body of propositions in question."[3]

These logical structures may be in fact the manifolds that Ludwig Wittgenstein had in mind in the Tractatus. I shall try to develop this idea next.

In his Principles of Mechanics Heinrich Hertz defines a dynamic model in the following manner. A material system (or a system of material points) is a dynamic model of another material system if and only if the two systems have:

a) the same number of coordinates of position;

b) the same equations of condition;

c) the same magnitude of displacement.

It should be pointed out that condition (b) is by far the most interesting. If the first is referring to the projection (Abbildung) of a physical system on a coordinate space, and the third refers to the conservation of displacement, the second one stipulates the existence of the same equations of condition in both systems. The question is: why could they be different? Let us think of some examples. Let us suppose that we want to model the trajectory of a physical system with two material points that move through space. According to Hertz the model would be characterized by:

a) some spatial coordinates;

b) assuming that the system contains "hidden masses", we will have to express the path of the two points by referring to their hidden masses in such a manner that their (geometrical) path in the configuration space will conform to the spatial displacement of the system described without the hidden masses;

c) a magnitude for displacement.

We could make this example even more intuitive focusing on condition (b). Let us think of the physical system formed by the Earth and the Moon. As we all know the planets of our solar system are situated at considerable distances from one another and that is why they can be represented in classical

[2] PR, 46,79.

[3] (Landini, 2003,108).

mechanics as material points (without mass) revolving around the Sun. For instance, the distance between the planet Mars and the Earth is big enough to neglect the gravitational attraction exerted and therefore we can represent them as material points in motion. However, the physical system formed by the Earth and the Moon cannot be described likewise because the two bodies are close enough as to exert observable gravitational effects one upon the other. This means that their masses are relevant to their motion. Yet, Hertz wants to reduce forces from classical mechanics and express gravitational effects in terms of free systems (like the Earth-Mars system from our example) with some hidden masses that would constrain internally the system's motion. In order to do that, he needs to introduce in his mechanics some new elements:

"It is always permissible to regard a system of material points as being composed of an infinite number of material particles." (H. Hertz, 2002, 46)

But what makes this formal trick permissible? Simply said, it is the equations of condition stating that the spatial displacement of the Earth-Moon, for instance, is equal to the geometrical path of a system with an infinite number of material particles in a configuration space. It is obvious that Hertz is referring here to the same kind of permissibility as is Ludwig Wittgenstein in Philosophical Remarks. Each physical body with mass can be regarded as an n-dimensional point in a geometrical manifold called the configuration space[4] as long as the spatial displacement and the geometrical path of the body are equal, or more philosophically put:

"... if we regard the condition of the model as the representation [my emphasis] of the condition of the system, then the consequents of this representation (...) are also the representation of the consequents which must proceed from the original object..." (Hertz, 2002, 177).

So such a formal trick is permissible only if it leads to correct predictions, and the key to making correct predictions lays, among other things, in finding the right equations of condition for the model, such as expressing displacement in terms of geometrical paths.

Here it seems that multiplicity (the number of coordinates of motion) is not established by phenomenological analysis as in von Helmholtz's epistemology, but more likely through some sort of a priori analysis of matter, focused on the conditions of mechanical representation of physical bodies. This point of view has been expressed by several authors interested in the epistemology that underlies Hertz's system of mechanics, and was also emphasized by Ludwig Boltzmann. Often cited is the following fragment from the Principles of Mechanics:

"The agreement between mind and nature may by (...) likened to the agreement between two systems which are models of one another, and we can even count for this agreement by assuming that the mind is capable of making actual dynamical models of things, and working with them" (Hertz, 2002, 177).

The "hidden masses" in Hertzian mechanics are in fact elementary positions in the configuration space (an n¬-dimensional manifold) to which a model associates semantically some generic material characteristics

[4] "... it [mass] can be thought of as divided into arbitrarily many equal mass-particles, each of which indestructible and able to serve as a characteristic in order to definitely and unambiguously coordinate one point in space with another point in space at another time." (Hertz 1953, 300).

and they are important in order to define a structure in Hertzian mechanics – the degrees of freedom of a physical system.

Let us go back to our example about the two physical systems discussed earlier: Earth-Mars and Earth-Moon. Stipulating that each system consists of n material particles moving in a three-dimensional Euclidian space, the Earth-Mars (moving without constraints) would be projected on a 3n configuration space, while the Earth-Moon, moving with constraints, would be projected on a 3n-k dimensional space. This means that the second system will have less degrees of freedom, given the fact that certain connections between particles are rigid (i.e. those corresponding to gravitational attraction). So, the multiplicity of a model for a physical system is not given by the number of units of matter that describe the system simpliciter, but by its structure, i.e. the degrees of freedom that the system possesses. Thus, even if in our initial "situation" we had observable effects of gravitational attraction, in the geometric Darstellung of the system, gravitational attraction is dealt away or eliminated by stipulating certain configurations of elementary material particles in an abstract space.

### 3. "Images" and "representations"

In the basic semantic view of how a proposition means something, propositions are considered "images" (Bilder) of facts in the following sense: there is a 1:1 correspondence between the elements of propositions and the elements of facts. It seems that Wittgenstein presents a clear model-theoretical approach to meaning in the Tractatus (Hacker, 1981). However, this interpretation hides a few traps. The author of the Tractatus assumes the existence of a primitive 1:1 semantic relation between propositional elements and elements of the world, but this relation is not interpreted extensionally in set-theoretic terms. Logical multiplicity expresses not a 1:1 correspondence between set-theoretic extensions and names, predicates etc., but the degrees of freedom or the structure that facts share with propositions in logical space with no reference whatsoever to extensions and types. The 1:1 correspondence refers to an isomorphism of models with generalized coordinates.

As we have seen in the previous section, the basic idea behind dynamic modeling is that multiplicity should express the number of freedoms that a system possesses in a state-space. To build a dynamic "scenario" is to make certain assumptions regarding how to organize the relevant information using coordinates, equations of conditions and n-dimensional vectors organized in such manifolds. Actually, this is the main function of a theory of representation (Darstellung) in Hertzian mechanics. Only after these aspects are settled, a dynamic model (understood as number of equations that define the temporal evolution of the system) could express the evolution of such n-dimensional vectors in respect to time.

In order to see the analogy between semantic analysis and dynamic modeling in Tractatus, I suggest going back again to our example with the two specific physical systems: Earth-Mars and Earth-Moon. The first one is a free system with 6 degrees of freedom. The second one is not a free system, it has only three degrees of freedom (because of the gravitational attraction that forms a rigid connection in the configuration space between Earth and Moon, and so the two material bodies move like a single point). However, the Earth-Moon can be treated like a free system with three degrees of freedom. So although it consists of two material particles, the system is mechanically equivalent to a single point and its displacement is analyzed as a three-dimensional vector. The same situation holds, as the author of the Tractatus points out, for propositions. We can have propositions like (P) "The sky is blue and the grass is green" and propositions like (P′) "The sky is blue and the sky is green". In the first case our proposition has four degrees of freedom. In the second case, the proposition has fewer degrees of freedom because of color exclusion, and so the analysis of (P) as: "p&q" is not a correct analysis (TLP 6.3751).

In order to get an elementary analysis we need to find an adequate Darstellung of (P′), and for this we need to take into consideration what Wittgenstein calls "the logical structure of color" (TLP 6.3751) or, keeping in mind the analogy with Hertzian mechanics, the "rigid connections" of colors. Thus, we need to deepen our analysis and dig for the structure of atomic/elementary propositions. This step into the analysis of atomic/elementary propositions is similar to the step taken by some logicians from first-order propositional logic to first-order predicate logic. However, the author of the Tractatus does not analyze properties (like color, for instance) as predicates, but prefers a Hertzian, eliminative approach in respect to them – a strategy undertook also by Bertrand Russell in his 1924 lecture on logical atomism (see section 1) It seems that atomic/elementary propositions as semantic "scenarios" of the world may be structured as n-dimensional manifolds by defining the degrees of freedom of elementary propositions in logical space, without making reference to logical types.

In another paper[5] I presented the formal details of a structural analysis of elementary propositions as n-dimensional manifolds. I argued that elementary propositions cannot be considered "images" (Bilder, Modelle) of states of affairs and vice versa unless we define a "parameterized representation" (Darstellung) in logical space of both elementary propositions and states of affairs.

Here, however, I choose to focus only on some general aspects of such a structural analysis, in order to get a more accurate reading of Ludwig Wittgenstein's Bilder and Darstellungen from Tractatus logico-philosophicus. By appropriating the method of generalized coordinates from classical mechanics, the elementary "parts" of a proposition (or, in short, the elementary propositions) can be defined formally as "dimensions" in an abstract n-dimensional space called a manifold. This explains why Wittgenstein takes elementary propositions to be logically independent. Nevertheless, elementary "parts" – just like Hertz's material points – have internal structure. Otherwise they could not be considered isomorphic to states of affairs.

In some particular cases, like (P), elementary "parts" resemble the material points in the Earth-Mars example – i.e. they are free. In other cases, like (P′), elementary "parts" resemble the material points in the Earth-Moon example – i.e. they hide combinatorial constrains. It is impossible to treat the elementary "parts" of (P′) as independent

[5] "The logical independence of elementary propositions in Tractatus logico-philosophicus" (in Romanian) Analele Universității București. Filosofie (2006): 153-164.

dimensions in propositional analysis. For a correct analysis of (P') it is necessary to dig out its complicated internal structure and establish the constraints.

Following the Hertzian analysis of material points into mass particles we may describe briefly the structural analysis with constraints in the following terms:

i) Formally, each proposition p can be analyzed into manifolds of the form Mn, where M is the topological base (i.e. the T-F base given that each proposition is either true or false), and n is the number of freedoms defined on M. A proposition p with n atomic/independent parts is a 2n manifold. A proposition with n-k atomic/independent parts is a 2n-k manifold. This can be expressed more perspicuously in the following manner:

a) given a proposition p with a T-F base (M) and n elementary parts, the corresponding manifold for n dimensions with 0 constraints would be Mn.

b) given a proposition p with a T-F base (M) and n elementary parts, the corresponding manifold for n dimensions with k constraints would be Mn-k

ii) Then, for each freedom r defined on M, we assume a corresponding elementary state of the world or a T/F value; and for each Mr we assume a corresponding matrix of elementary states of the world or a matrix of T/F values.

In this manner, any logical representation Mn of a proposition is a n-dimensional manifold with 0 or 0 < k constraints. In other words, the manifold of a complex proposition is dependent upon the T/F combinations of elementary propositions. However, the only explanation for the fact that in some cases we have constraints upon the T/F combinations of elementary propositions is that elementary propositions have internal structure.

Two observations:

a) M is not a set; it is a topological basis for representing (darstellen) elementary states/ propositions in logical space.

b) n expresses the freedoms of a proposition/fact in logical space, determined by the constraints k applied on the base M.

To illustrate these formal aspects I will now return to the discussion from the first part of this section. Thus, following a simple analysis of color predicates, the example I chose above, we see that logical structure of color makes certain combinations of truth-values rigid. That is why the logical form of (P') may by in fact something more complicated than "p&q"[6] . The logical form of (P') is more likely, as Wittgenstein suggests (TLP 6.3751) , "p& ~p":

| p | ~p | p&~p |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |

(D)

(P') is a proposition excluded by the logic of color. According to our analysis, (P') may be described a 2-dimensional manifold of truth-values (therefore by two degrees of freedom). This is because the second "part" of (P') represents a dimension dependent upon the first "part". Thus, instead of having four degrees of freedom, (P') has only two.

Unlike (P'), (P) can be described by a 4-dimensional manifold of truth-values (1;1) (0;1), (1;0), (0,0), because all combinations are

[6] "Only when we analyze phenomena logically shall we know what form elementary propositions have. (...) The logical structure of elementary propositions need not have the slightest similarity with the logical structure of propositions. Just think of the equations of physics – how tremendously complex their structure is. Elementary propositions too, will have this degree of complexity" (WVC, 42).

permitted by the logic of color.

| p | q | p&q |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 0 | 0 | 0 |

(D1)

(D) and (D1) are Bilder of propositions (P) and (P′) in logical space. Their form is dependent upon giving the adequate Darstellung for (P) and (P′), i.e., upon finding the right number of freedoms (or the parameter r) of the situations described by (P) and (P′). In the case of a free model (with no constraints), r = n (the maximum number of possible T/F combinations for conjunction). In the case of a model with rigid connections or constraints, we should have r < n.

However, it is not really clear whether in the Tractatus Wittgenstein accepted that elementary states have indeed structure – although he refers to "configurations of objects" (TLP, 2.01, 2.0272) as an analysis for elementary states of the world, quite similarly to the manner in which Hertz refers to "configurations of mass particles" as an analysis for material points with mass. As we have seen in the previous section, mass particles do not measure mass, but they only express the dependence of the path of material points in the configuration space upon mass. Mutatis mutandis, the Tractarian objects are not objects per se, but rather they express the dependence of states of affairs, and consequently of propositions, upon their internal structure, i.e. upon what these states actually are: colors, sounds, time etc. Each spatial, temporal or color configuration leaves a space of 0 ≤ r ≤ n dimensions or degrees of freedom for combinations. For instance, no fact can be at the same time two different colors or two different sounds, but it can be at the same time a position and a color, a color and a sound etc. The parameter r expresses such dependencies in logical space in terms of freedoms and constraints on base manifolds (M), without analyzing physical properties (TLP, 2.0231)

From this perspective that assigns structure to elementary propositions and states, we can also get a better grasping of Wittgenstein's own critique of logical analysis, presented in "Some Remarks on Logical Form" (1929). There, he starts from the observation that a correct logical formalization of (P′) is not "p&~p" because (P′) does not express a contradiction, as he believed in the Tractatus, but only a false statement. "The sky is green" is not equivalent to denying "The sky is blue". In fact, it is possible to have a situation in which the sky is neither blue, nor green, but a sort of dark gray. And for this we need a different logical analysis of color and, more generally, a different way to express dependencies of states/propositions upon their content. The main reason why the type of analysis from the Tractatus fails is that it cannot formalize properly propositions like (P′). While (P) has four degrees of freedom, (P′) has only three. Most of the argument from RLF runs in the direction of showing that: (i) the second "part" of (P′) cannot be analyzed as a rigid dimension, i.e. as dependent upon the first dimension of (P′) as in "p&~p": we cannot obtain "The sky is green" by applying negation to "The sky is blue" because between the two colors there is a difference of degree that cannot be caught by the logical formalism of negation. However: (ii) the two colors are not entirely independent, and (P′) cannot be formalized as "p&q", so the analysis from the Tractatus must fail in some respect.

Wittgenstein had in mind certain basic postulates regulating the behavior of argument-places for color when he discussed "the logic of color". Thus, even if we cannot obtain straightforwardly "The sky is green" from denying "The sky is blue", we can generate a contradiction in the following manner. Let a be an object and '…' indicate other argument-places which may be form indicators of the object (spatiotemporal position, etc.) and c its color. The postulate would be:

(CP) If Pa…c… then for every x which is a color (i.e. fills the argument-place of a color) distinct from c, then ~Pa…x… .

Let "The sky is blue" be Pa…b… Now, by (WC), if Pa…b… then for every color x distinct from b, ~Pa…x…. Now, suppose that the sky is blue and green: Pa…b…& Pa…g…. Two applications of simplification, a modus ponens, an introduction of conjunction are all it takes to obtain the contradiction. What Wittgenstein was trying to say in "Some Remarks on Logical Form" was that Pa…b… and Pa…g… are contraries: they cannot be both true but they can be both false. This is a clear consequence of (CP).

But would not this imply that all propositions with a difference in their composition contradict each other in this sense? Not necessarily. For example, take Px1,…, xn and Px1,…, xm to mean "This very person in this very spatiotemporal position and with this very color and so on is sitting on the couch" and "This very person in this very spatiotemporal position and with this very color and so is watching TV", respectively. There is only a contradiction when there is a difference in the argument-place fillers when these constitute part of the form of the object.

The problem with this analysis at the propositional level is that there is no finite means to express (CP) in full generality, i.e. for the form of any object, although it can be done for particular cases. The analysis from the Tractatus fails because Wittgenstein's assumption that the relation between the propositional and the object levels is straightforward fails. The main problem is that Wittgenstein never specified what should be the number of argument places of a proposition in general; it is what is missing to give an appropriate use of Hertz's idea and calculate freedom more accurately.

**4. Final remarks**

Understanding how the human mind reasons and how reasoning influences behavior represents very important topics in the field of the social sciences today (like economics) not only from a theoretical perspective, but also from a practical one. Such understanding could be useful in order to assess and solve, for instance, leadership issues, i.e. by determining the breadth and limits of cooperative behavior inside organizations, or by explaining how collective behavior emerges from individual behaviors in different social groups etc. In this paper I focused on a historical aspect concerning an important step in the development of modeling human reasoning: the suggestion from late 19th century's epistemology of science to associate the modeling of physical phenomena with logical modeling, in order to figure out what human reasoning is really about. As we can easily see from this study, the endeavor was troublesome and prone to severe difficulties, even when seeking to analyze reasoning about simple things such as the color of objects. Nevertheless, this scientific approach to representation and inference managed to raise a larger interest for logical modeling and it led over time to one of the most effective tools that we currently have in modeling both human reasoning and behavior: i.e. game-theory, a very sophisticated abstract tool that Ludwig Wittgenstein's early attempts could not really anticipate.

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