The Determination of the Production Function's Stochastic Frontier at the Industry Level

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Abstract: *For this analysis, the first step consists in estimating the stochastic frontier of the production function.*

This study is designed to analyze the metallurgic industry, with the determination of the production function's stochastic frontier at the industry level.

Analyzing the results of the implementation of the above models we observe that some companies have a high efficiency on the whole range, and some companies have a low efficiency in this period.

Keywords: production's stochastic frontier, functions of production, industry, ranking companies.

1. Estimating the production function's stochastic frontier

Aigner, Lovell and Schmidt (1977) and Meeusen și van den Broeck(1977) have independently proposed the production function's stochastic frontier where an addi - tional random error, v_i, is added to the non-negative random variable, u_i, in the following equation of the model proposed by Aigner and Chu (1968).

$\ln(y_i)=x_i\beta-u_i$, i=1…n, where:

x_i is the line (k-1) of the vector, whose first element is 1, the other elements are logarithms of the quantities of the input K used by i company;

$\beta=(\beta_0, \beta_{1,...}\beta_k)$, is the vector's (k-1) co - lumn of unknown parameters that will be estimated;

u_i is a non-negative random variable, associated with technical inefficiency in production of the firms from the industry analyzed.

Considering the parameter estimation of production function's frontier starting from the formula of Codd – Douglas using

data from n companies in order to provide:

$(y_i)=x_i\beta+v_i-u_i$, i=1…n

The random error v_i, justifies the errors measurement and other random factors such as the effects of weather, luck etc. , in the value of the output variable and the combined effects of the input unspecified variable in the production function. Aigner, Lovell and Schmidt (1977) thought that v_i were independently and identically distributed to the random normal variables with zero average and constant variation, σ_v^2, independent of u_i, which were considered to be independently and identically exponential distributed or random half normal variables.

The model defined by this equation is called Production function's stochastic frontier because the output values are higher edged by the stochastic (random variables, $\exp(x_i\beta+v_i)$, random error v_i can be positive or negative and also the stochastic frontier of the outputs varies around parts that define the frontiers's model $\exp(x_i\beta)$).

The deterministic component of the Frontier model $y=\exp(x\beta)$, is extracted assuming that the result is reduced proportionally. The outputs and the inputs observed for two firms, y and j are presented in the graphic. The i firm uses the level of input x_i to produce the y_i output. The input-output amount value observed is indicated by the marked point x above the x_i value. The output value $y_i^*= \exp(x_i\beta+v_i)$ is marked by x surrounded over the production function due to the random error, v_i , is positive. Similarly, j firm uses the input level x_j and produces y_j . However the Frontier output, $y_j^*= \exp(x_j\beta+v_j)$ is under the production function v_j is negative. Of course, the Frontier stochastic outputs y_i^* and y_j^* are not observed because of the random error, v_i and v_j are not observable. However the Frontier stochastic model's deterministic part is observed as being between the Frontier stochastic outputs. The observed outputs can be higher than Frontier deterministic part if the random errors are bigger than the corresponding inefficiency effects ($y_i> \exp(x_i\beta+v_i)$ if $v_i>u_i$).

The stochastic frontier's model pemits the estimation of standard errors and testing hypotheses using the traditional method of Maximum Probability.

2. Metallurgic industry. Presentation data

The application uses data from the metallurgic industry between 2005 and 2007.

As development, the metallurgical production structure between 1990 and 2007 had a great rise. Being given these values of progress, results the chart in figure 2, which represents the increase of metallurgical industry structures during the time taken as an example.

1990	1991	1992	1993	1994	1995	1996
99,2	265,9	712,5	1587	4184,6	6254,4	9649,6

1997	1998	1999	2000	2005	2006	2007
25180,7	25367,5	35270,3	72322	128119,1	152542,3	171807,2

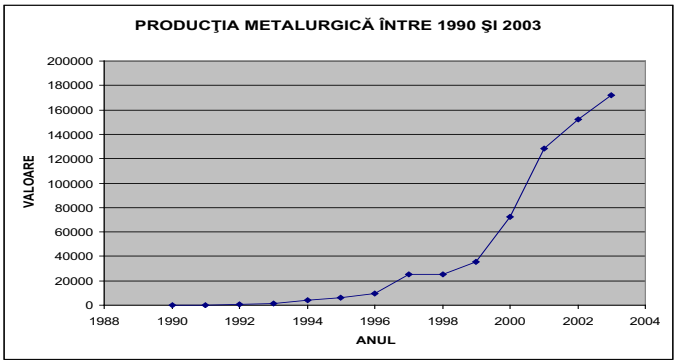

Figure 1

The application establishes the efficient and the inefficient firms in the metallurgic industry. A few examples show the best way to describe the means for using the Frontier programme. In this section we will consider the estimation of:

1) the Cobb-Douglas production frontier using crossed data and a half normal distribution.
2) the Battese and Coelli specification (1992)

We have gathered data from the activity reports concerning the following firms that are part of the metallurgic industry, covering a three year time period: 2005, 2006 and 2007. All these firms taken as an example are quotable on the Rasdaq Electronic Stock Exchange at the present moment. The source for these activity reports is the website of the Rasdaq Electronic Stock Exchange. The list of these firms is as following:

- CILINDRUL SA-CĂLAN, HUNEDOARA
- DAN STEEL GROUP SA-BISTRIŢA NĂSĂUD
- DUCTIL SA-BUZĂU
- ELECTROCRBON SA-SLATINA
- ELSID SA-DAMBOVIŢA
- GRANTMETAL SA-BUCHAREST
- GRIVIŢA SA-BUCHAREST
- INTFOR SA-GALAŢI
- LAMINORUL SA-ROMAN
- LAROMET SA-BUCHAREST
- MATRIŢA SA-ODORHEIUL SECUIESC
- MECANICA 94 SA-DROBETA TURNU SEVERIN
- MECHEL SA-CÂMPIA TURZII
- METALURGICA SA-HARGHITA,BIHOR
- METALURGICA SA-REGHIN
- METALURGICA SA-VLAHIŢA
- MITTAL STEEL HUNEDOARA SA
- NEFERAL SA-BUCHAREST
- OŢELURI PENTRU SCULE SA-HARGHITA
- SATURN SA-ALBA
- SOMETRA SA-COPSA MICA
- TUFON SA-CRAIOVA
- TURNATORIA CENTRALĂ ORION-CÂMPINA
- TURNSEV SA-DR. TURNU SEVERIN

We used as input:

The number of employees - the firm's labour force in the respective year.

Fixed assets- goods and stock that are used for a longer time period in the activity of the patrimonial unit. These are not wasted

on the first use and the accountancy has divided them into three categories: tangible assets, intangible assets and financial assets.

Stocks are assets:

That are held to be sold during the proceeding of the ordinary activity

In the progress of production for a future sell; or

Such as raw material, material and other consumptible articles that are to be used in the production process or for carrzing out services

We used as output:

Operating income which includes:

- income from selling products, commodities, executed works and carried out services
- income from the production in stock, which means the surplus or the deficit in the difference between the effective production value of the products in stock and the production in progress at the end of the period on the one hand and on the other hand the value of the initial products in stock and the production in progress, without taking into account the provisions for depreciation set up for these elements;
- income from the production of fixed assets, which means the cost of the works and the expenses undergone by the patrimonial unit for investments, which are registered as tangible or intangible assets;
- income from operating subsidies that stand for subsidies received in order to cover price differences and losses, as well as other subsidies that the patrimonial unit benefits of coming from the state or other patrimonial units;
- other current operating incomes, which include income from recovering outstanding debts and other operating income;
- income advanced as revenues or outstanding debts for undelivered goods, works or labour conscription not carried out, which are not considered income of the accounting period, being registered in accountability in a separate account of the balance sheet..

Turnover is the sum of the commodities and production sale, at a selling price, excluding price reduction granted to the client and Value Added Tax. It is an indicator used for describing the size of the company and for analysing its economic-financial status. We can find the turnover in the company's profit and loss account

3. Study case

We used data from the annual reports of the 24 firms presented before and we fol - lowed a few examples from the file attached to the Frontier programme, examples that use the models presented above:

In order to resume the example we as - sume three inputs: the number of employees, fixed assets and stocks, and as outputs: operating incomes and turnover in all the cases. In the crossed examples we will have 24 firms, whereas in the examples of the data tables- 24 firms and we will use 3 time periods.

1) The Cobb-Douglas production frontier using crossed data and assuming a half normal distribution.

In the first example the Cobb-Douglas production frontier is estimated

$$ln(Q_i) = \beta_0 + \beta_1 ln(K_i) + \beta_2 ln(L_i) + (V_i - U_i),$$

where Q_i, K_i şi L_i are inputs, capital and labour, and V_i and U_i are assumed to be normal and half normal distributed.

From data analysis, we conclude that some firms have a high efficiency, that is a value in the interval [0.8,1], and some have a low efficiency somewhere between [0,0.3].

The table of the firms with a high efficiency

DAN STEEL GROUP SA-BISTRITA NASAUD	0.877
INTFOR SA-GALATI	0.870
MATRITA SA-ODOwRHEIUL SECUIESC	0.843
MECHEL SA-CAMPIA TURZII	0.954
METALURGICA SA-VLAHITA,HARGHITA	0.820
MITTAL STEEL HUNEDOARA SA	0.982
SATURN SA-ALBA	0.860
TURNATORIA CENTRALA ORION-CAMPINA	0.918

The table of the firms with a low efficiency

CILINDRUL SA-CALAN,HUNEDOARA	0.252
GRANTMETAL SA-BUCHAREST	0.272
GRIVITA SA-BUCHAREST	0.282
LAROMET SA-BUCHAREST	0.211
MECANICA 94 SA-DROBETA TURNU SEVERIN	0.249
METALURGICA SA-HARGHITA,BIHOR	0.180
TURNSEV SA-DROBETA TURNU SEVERIN	0.146

2) The Battese and Coelli specification (1992)

Results obtained by processing data using the Frontier programme:

The table of the firms with a high efficiency (in the interval [0.8,1]):

Firm	Year	Efficiency
MECHEL SA-CAMPIA TURZII	1	0.952
MITTAL STEEL HUNEDOARA SA	1	0.969
MECHEL SA-CAMPIA TURZII	2	0.943
MITTAL STEEL HUNEDOARA SA	2	0.904
MECHEL SA-CAMPIA TURZII	3	0.916
MITTAL STEEL HUNEDOARA SA	3	0.824

The table of the firms with a high efficiency (in the interval [0.8,1]):

MECANICA 94 SA-DROBETA TURNU SEVERIN	1	0.291
METALURGICA SA-HARGHITA,BIHOR	1	0.280
GRIVITA SA-BUCHAREST	2	0.294
MECANICA 94 SA-DROBETA TURNU SEVERIN	2	0.274
METALURGICA SA-HARGHITA,BIHOR	2	0.276

TURNSEV SA-DROBETA TURNU SEVERIN	2	0.263
GRIVITA SA-BUCHAREST	3	0.298
MECANICA 94 SA-DROBETA TURNU SEVERIN	3	0.275
METALURGICA SA-HARGHITA,BIHOR	3	0.278
TURNSEV SA-DROBETA TURNU SEVERIN	3	0.263

4. Conclusion

Analysing data obtained by applying the above models, we can see that some firms have a high efficiency during the entire interval, whereas some firms have a low efficiency during this interval. Among the ones with a high efficiency we can name: *Mechel SA – Câmpia Turzii* and *Mittal Steel Hunedoara SA*, and from the ones with a low efficiency the following firms:: *Mecanica 94 SA – Drobeta Turnu Severin, Griviţa Sa – Bucharesi, Turnsev SA – Drobeta Turnu Severin* şi *Metalurgica SA – Harghita Bihor*

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