Attitudes towards risk and insurances

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Abstract: *The paper presents the attitudes towards risk and their influence in insurance theory. Based on these attitudes, the insurance companies can distinguish between agents with inclinations towards risks and the ones with aversion towards risk, who interest them.*

Key words: risk, attitudes towards risk, insurance, Arrow-Pratt indices.

Among the different ways of measuring attitudes towards risk, the Arrow-Pratt indices are the most used in the scientific literature. The paper will present the most important ways of measuring the attitudes towards risk as well as the advantages and disadvantages that they generate.

1. Absolute and relative risk aversion – coefficients

We shall consider two von Neumann-Morgestern utility functions U_1, U_2 defined on $\Re$, monotone increasing, strictly concave and of a C^3 class (al least three times differentiable). The *risk aversion coefficients* are defined as follows:

Definition 1. Absolute Risk Aversion. U_1 shows a strictly larger risk aversion in Arrow-Pratt sense if its *absolute risk aversion coefficient* is larger:

$$ra_1(x) = -\frac{U_1''(x)}{U_1'(x)} \geq ra_2(x) = -\frac{U_2''(x)}{U_2'(x)} \quad \forall x \qquad (1)$$

Definition 2. Another classification is based on the **relative risk aversion coefficient** that represents the elasticity of the marginal utility respect to the wealth

$$r_1(x) = -x\frac{U_1''(x)}{U_1'(x)} \geq r_2(x) = -x\frac{U_2''(x)}{U_2'(x)} \quad \forall x \qquad (2)$$

These coefficients are local measurements of risk aversion excepting the cases of utility functions with *constant risk aversion* for $\beta > 0$, such as: $U(x) = \alpha - \beta e^{-ax}$, where a is the absolute risk aversion coefficient.

2. Pratt Theorem

In 1964, Pratt created a risk aversion measurement that can also have some flows. For instance, any classic economic problem is about the risk aversion agent's willingness to pay for insurance. If, economically speaking, it is necessary to measure the risk aversion, and then an agent with a larger risk aversion will be willing to pay more for an insurance package. The insurance premium is defined as the maximum availability to pay in order to avoid to get a lottery X with an expected value of E[x]=0. So, the insurance premium π will actually be the certainty equivalent of the lottery X, which satisfies the following condition:

$$E[U_i(w+X)] = U_i(w-\pi_i), \quad i \in \{A, B\} \qquad (3),$$

where w is the initial wealth.

Pratt Theorem. The following three conditions are equivalent:

$$-\frac{U_1''}{U_1'} \geq -\frac{U_2''}{U_2'}, \quad \forall x \qquad \textbf{(i)}$$

$\exists G$ - A monotone increasing transformation, $G' \geq 0, G'' \leq 0, U_1 \equiv G(U_2)$ **(ii)**

$$\pi_A \geq \pi_B, \quad \forall w \qquad \textbf{(iii)}$$

a. Strong Absolute Risk Aversion

Because the failures of associating a larger risk aversion to a larger insurance premium, Ross gave a new definition: the *strong absolute risk aversion*.

Definition 3 (Ross). U_A shows a stronger absolute risk aversion than U_B if:

$$(\exists \lambda > 0)\ \frac{U_1''(x_1)}{U_2''(x_1)} \geq \lambda \geq \frac{U_1'(x_2)}{U_2'(x_2)} \quad (\forall x_1, x_2)$$

This new approach is strictly more powerful than the Arrow-Pratt measure.

Proposition 1. If U_A shows stronger absolute risk aversion than U_B, then it also shows a larger absolute risk aversion but the reciprocal doesn't hold.

b. Risk Aversion that depend on wealth

But, most of the times, the risk aversion is dependent of the investor's wealth. This is why, it is expected that the wealthier individuals are more inclined towards risk than the others. This leads to the assumption of decreasing risk aversion.

Definition 4. The utility function *U(x)* shows a *decreasing absolute risk aversion* (**DARA**) if the following condition is satisfied:

$$-\frac{U''(x+y)}{U'(x+y)} \geq -\frac{U''(x)}{U'(x)} \quad (\forall x, y > 0)$$

and it shows a *strong decreasing absolute risk aversion* if:

$$(\exists \lambda)\ \frac{U''(x+y)}{U''(x)} \leq \lambda \leq -\frac{U'(x+y)}{U'(x)} \quad (\forall x, y > 0)$$

Analogous, the increasing absolute risk aversion (**IARA**) can be defined if:

$$-\frac{U''(x+y)}{U'(x+y)} \leq -\frac{U''(x)}{U'(x)} \quad (\forall x, y > 0)$$

as well as an increasing *stronger* absolute risk aversion if:

$$(\exists\lambda) \quad \frac{U''(x+y)}{U''(x)} \geq \lambda \geq -\frac{U'(x+y)}{U'(x)} \quad (\forall x, y > 0)$$

Proposition 2. (Equivalent Conditions for DARA) The utility function *U(x)* shows a decreasing absolute risk aversion if and only if:

$$\frac{U'''(x)}{U''(x)} \geq \frac{U''(x)}{U'(x)} \quad (\forall x)$$

A necessary condition is that $U''' < 0$.

If the relation between the risk aversion and the risk premium or the demand for riskier assets is not very clear, then it can be said that the wealthier agents have more risky assets if the risk aversion is decreasing in wealth. We shall study the influence of the attitude towards risk over the Insurance Theory

3. Attitudes towards risk

A risky situation or a lottery is a situation where the possible outcomes are associated to a certain state of the nature that will take place with certain probabilities. The probabilities sum is 1. For instance, we shall consider a lottery L that has equal probabilities to get the outcomes x_1 and x_2 where $x_1 < x_2$. The expected income (expected monetary value) from the lottery is $E(L) = \frac{x_1 + x_2}{2}$.

The attitude towards risk of an individual can be determined. The individual is questioned if he is willing to invest in a lottery L or if he prefers to receive a certain amount of money, *Ec* named ***certainty equivalent***. This represents the individual's benefit that gives him a satisfaction equal to the mean satisfaction from the lottery: *U(Ec)=E[U(L)]* , where U is the utility function associated to the lottery L.

Comparing the certainty equivalent to the expected monetary value of the lottery, we can get three attitudes towards risk:

a) An individual who is indifferent between the amount of money that he can receive with certainty and the expected monetary value of the lottery is **risk neutral**.

b) An individual who prefers to invest in the lottery instead to accept the certainty amount of money, *Ec*, is considered to be **risk loving.**

c) The individual that prefers the opposite situation is **risk averse**.

If we assume that the individuals compute the expected utility of the lottery as a weighted mean of the possible outcomes, then there is a strong connection between the attitudes towards risk and the marginal utility of the individual's wealth.

In the case of the risk aversion, we get the situation where:

$$\frac{U(x_1) + U(x_2)}{2} < U\left(\frac{x_1 + x_2}{2}\right)$$

which means that the mean of the outcomes utilities is smaller than the utility of expected income. So, if the economic agent has aversion towards risk, then the following inequality will take place: $U[E(L)] > E[U(L)]$. Using the notion of certainty equivalent, the inequality can also be written as: *U(Ec)=E[U(L)]*.

But, we know that the utility function U is strictly increasing, then *Ec<E(L)*, which means that the individual prefers to receive

less money but he is certain that he can receive this amount.

4. Risk aversion and insurances

If the individuals are risk averse, then they will be willing to get an insurance against the risk. Because most individuals buy insurances against large risks, such as houses or car insurances, we can assume that most individuals are risk averse. These individual can act as individuals with an inclination towards risk: they gamble, the go to casinos, etc. But, these activities can be only for fun and they imply a small amount of money that can be lost. These individuals are actually risk averse when it is about large amount of their money.

Insurance demand

We shall consider a risk adverse individual that owns an expensive property (a house or a car). This property is subject to the risk of being stolen. The individual wealth is T and the value of the car is C. Then, the probability for the car to be stolen is p. We assume that p is exogenously given (it doesn't depend on the individual's actions and it is known by both parties in the insurance: the insurant and the insurance company. Obviously, this is a case of symmetric information.

Then, the problem that the individual is faced with is represented in Figure 1, where we represent the insurant's utility function depending on the wealth. We shall assume the situation that the individual encounter if he is doesn't buy the insurance. If the car isn't stolen, the individual's wealth will be T and his utility is $U(T)$ and when the car is not stolen, the individual's wealth is $T-C$ and his utility is $U(T-C)$. His expected wealth will be $E(W)$:

$$E(W) = p(T-C) + (1-p)T \qquad (4)$$

And the expected utility is $EU(N)$, where N is the un-insurance situation:

$$EU(N) = pU(T-C) + (1-p)U(T) \qquad (5)$$

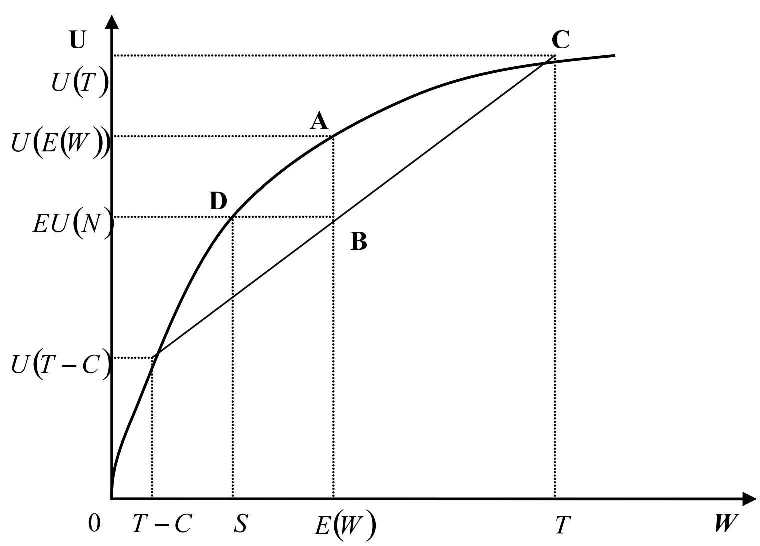

Figure 1. *Risk aversion and insurances*

Because this individual is risk adverse, then his expected utility in the risky situation, $EU(N)$, is smaller than the utility $U[E(W)]$ which he will get for a certain level of wealth equal to the expected wealth $E(W)$; in Figure 1, this situation is described by B, that is situated under A.

We can se that the wealth S will certainly lead to the same utility level $EU(N)$. So, if an insurance company decided to replace the insurer's car or to pay him a compensation Y, equal to the value of the car, then if the car is stolen, the insurance company will pay an insurance premium X as long as $X < T - S$ because it will guarantee a revenue of S or larger.

When the compensation paid by the insurance company to the client when the car is stolen is high enough to fully compensate him, then the insurance company will provide a *full insurance* that will be the subject of the following analysis.

When an individual is fully insured, he becomes certain of his wealth and if the car is not stolen, then his wealth is $T - X$ and if his car is stolen, his wealth will still be $T - X$ because the loss of his car is fully insured. As long as $X < T - S$, the individual has more wealth than S due to the payment of the premium in order to buy the insurance. Because $U(S) = EU(N)$, for a wealth larger than S, the utility is larger than the expected utility when there is no insurance and this is the reason for the individual to buy the insurance. If the insurance premium is $X > T - S$, for a compensation level of C, the individual would prefer to remain uninsured as long as his expected utility when he is not insured would be larger than a certain utility level when he is insured.

An individual that buys a full insurance receives a premium X for an uncertain payment from the insurance company. The compensation of the payment received from the insurance company is 0 with probability 1-p (if the car was not stolen) and C with probability p (if the car was stolen). Thus, we can say that the insurer takes a risk because he makes a certain payment for an uncertain compensation.

Insurance supply

We shall consider the insurance premium required by the insurance company. We assume that an insurance company is risk neutral and that the insurance market is a competitive one. Thus, the insurance company is in a competition with other companies in order to attract clients by reducing the insurance premium to a level that will bring it al least zero profit. Also, the operation costs are assumed to be zero. At the equilibrium point, for a full insurance, the premium is given by:

$$X = pC \quad (6)$$

From the equation 6, we can see that the insurance company receives the premium X with certainty and it pays the compensation Y, that is assumed to be equal to C with a probability *p*. The profit will be zero. In this case, the insurance is called a fair chances insurance; if $X > pC$, the insurance is an unfair chances one and if $X < pC$, then the insurance is one with favorable chances. This is why the equation 6 is also called fair chances constraint.

The individual's certain level of wealth, after he buys a full insurance is $T - pC = E(W)$, that is equal to the expected wealth when the individual is not insured. But, $E(W) > S$, then the insured's utility increases after he buys a full insurance at fair chances and the utility level is $U[E(W)]$, that will be larger than the utility when he is not insured, $EU(N)$. Then, the premium is $X = pC < T - S$, which means that the sum of money that he could pay and be indifferent between buying and not the full insurance.

Starting with this type of analysis, things tend to be more complicated. This is the case of finding the optimum contract that can be offered to insurants, as well as different other type of insurances or different types of risk measures (for instance, Yaari measure) and their impact in the insurance theory.

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